MACHINES

A machine is a device used to simplify work.
When using a machine, a force is applied at one point to overcome another force at another point.
A machine is used to;

- Convert energy from one form to another.
- Amplify the force used.

PRINCIPLE OF MACHINES:
It states that a small force applied (effort) moves a large distance to produce a bigger force that moves a load through a small distance.

TERMS USED IN MACHINES

Effort (E):
This is the force applied at one point of a machine to overcome the load.
The SI unit is the Newton (N).

Load (L):
This is the force which is overcome by the effort.
The SI unit is the Newton (N).

Mechanical Advantage (M.A):
This is the ratio of load to effort.

$$Mechanical\ Advantage = \frac{Load}{Effort}$$

$$\boldsymbol{M.A = \frac{L}{E}}$$

Mechanical Advantage doesn't have units.

Velocity Ratio (V.R):
This is the ratio of distance moved by the effort to distance moved by the load in the same time.

$$Velocity\ Ratio = \frac{Distance\ moved\ by\ Effort}{Distance\ moved\ by\ the\ load}$$

$$\boldsymbol{V.R = \frac{D_E}{D_L}}$$

Velocity Ratio doesn't have units.

Work input (W.I):
This is the work done by the effort to overcome the load.

OR

This is the product of effort and the distance moved by the effort.

$$Work\ input = Effort \times Distance\ moved\ by\ the\ Effort$$

$$\boldsymbol{W.I = E \times D_E}$$

The SI unit of work input is a joule (J).

Work output (W.O):
This is the work done by the machine to overcome the load.
OR
This is the product of load and the distance moved by the load.

$$Work\ input = Load \times Distance\ moved\ by\ the\ Load$$
$$\boldsymbol{W.O = L \times D_L}$$

The SI unit of work output is a joule (J).
Work output is also referred to as work done on the load.

Energy wasted:
This is the difference between Work input and Work output.
$Energy\ wasted = Work\ input - Work\ output$

Efficiency (η):
This is the ratio of work output to work input of a machine expressed as a percentage.

$$Efficiency\ (\eta) = \frac{Work\ output}{Work\ input} \times 100\%$$
$$Efficiency\ (\eta) = \frac{L \times D_L}{E \times D_E} \times 100\%$$
$$Efficiency\ (\eta) = \frac{L}{E} \times \frac{D_L}{D_E} \times 100\%$$
$$But\ M.A = \frac{L}{E}\ \ and\ \ \frac{1}{V.R} = \frac{D_L}{D_E}$$
$$Efficiency\ (\eta) = M.A \times \frac{1}{V.R} \times 100\%$$
$$\boxed{\boldsymbol{Efficiency\ (\eta) = \frac{M.A}{V.R} \times 100\%}}$$

NOTE:
In practice, the efficiency of a machine is always less than 100%.

Reasons why efficiency of a machine is always less than 100%:
- Some energy is wasted or lost in overcoming friction between the movable parts of a machine.
- Some energy is wasted or lost in lifting useless loads e.g. strings in pulleys.

How to increase the efficiency of a machine:
- By lubricating the movable parts of the machine i.e. oiling and greasing.
- By using light materials for useless loads.

Examples:
1. An effort of $200N$ moves a distance of $1.5m$ to lift a load of $480N$ through 1m. Calculate;
   (i) Mechanical Advantage.
   (ii) Velocity ratio.
   (iii) Work output.
   (iv) Work input.
   (v) Efficiency.

$E = 200N, \quad D_E = 1.5m, \quad L = 480N, \quad D_L = 1m$

(i) Mechanical advantage

$$M.A = \frac{L}{E}$$
$$M.A = \frac{480}{200}$$
$$M.A = 2.4$$

(ii) Velocity ratio

$$V.R = \frac{D_E}{D_L}$$
$$V.R = \frac{1.5}{1}$$
$$V.R = 1.5$$

(iii) Work output

$$W.O = L \times D_L$$
$$W.O = 480 \times 1$$
$$W.O = 480J$$

(iv) Work input

$$W.I = E \times D_E$$
$$W.I = 200 \times 1.5$$
$$W.I = 300J$$

(v) Efficiency

$$\eta = \frac{M.A}{V.R} \times 100\%$$
$$\eta = \frac{2.4}{1.5} \times 100\%$$
$$\eta = 1.6 \times 100\%$$
$$\eta = 160\%$$

2. If a lever is used to overcome a load of 50N by applying an effort of 10N. Find;
   (i) Mechanical advantage of the lever system.
   (ii) Efficiency of the system if the velocity ratio is 6.

$L = 50N, \quad E = 10N, \quad V.R = 6$

(i) Mechanical advantage

$$M.A = \frac{L}{E}$$
$$M.A = \frac{50}{10}$$
$$M.A = 5$$

(ii) Efficiency

$$\eta = \frac{M.A}{V.R} \times 100\%$$
$$\eta = \frac{5}{6} \times 100\%$$
$$\eta = 83.3\%$$

3. In a machine, 50N are used to overcome a load of 20kg. If the 20kg load moves a distance of 5cm whenever the 50N moves a distance of 25cm.
   Calculate;
   (i) Mechanical advantage. (ii) Velocity ratio (iii) Efficiency.

$E = 50N, \quad D_E = 5cm = \frac{25}{100} = 0.25m,$

$L = mg = 20 \times 10 = 200N, \quad D_L = \frac{5}{100} = 0.05m$

(i) Mechanical advantage

$$M.A = \frac{L}{E}$$
$$M.A = \frac{200}{50}$$
$$M.A = 4$$

(ii) Velocity ratio

$$V.R = \frac{D_E}{D_L}$$
$$V.R = \frac{0.25}{0.05}$$
$$V.R = 5$$

(iii) Efficiency

$$\eta = \frac{M.A}{V.R} \times 100\%$$
$$\eta = \frac{4}{5} \times 100\%$$
$$\eta = 0.8 \times 100\%$$
$$\eta = 80\%$$

4. An effort of 100N is used to raise a load of 200N. If the effort moves through a distance of 4m, calculate;
   (i) Distance moved by the load if the velocity ratio is 5.
   (ii) Energy wasted by the machine.
   (iii) Efficiency of the machine.

$E = 100N, \quad D_E = 4m, \quad L = 200N, \quad D_L = ?, \quad V.R = 5$

(i) Distance moved by load

$$V.R = \frac{D_E}{D_L}$$
$$5 = \frac{4}{D_L}$$
$$D_L = \frac{4}{5}$$
$$D_L = 0.8m$$

(ii) Energy wasted.

Work input:

$$W.I = E \times D_E$$
$$W.I = 100 \times 4$$
$$W.I = 400J$$

Work output:

$$W.O = L \times D_L$$
$$W.O = 200 \times 0.8$$
$$W.O = 160J$$

$$Energy\ wasted = W.O - W.I$$
$$Energy\ wasted = 400 - 160$$
$$Energy\ wasted = 240J$$

(iii) Efficiency

$$\eta = \frac{W.O}{W.I} \times 100\%$$
$$\eta = \frac{160}{400} \times 100\%$$
$$\eta = 0.4 \times 100\%$$
$$\eta = 40\%$$

5. In a machine which is 75% efficient, an effort of 300N is used to lift a load of 900N. If the load is moved through a distance of 2m, find the;
   (i) Mechanical advantage.
   (ii) Velocity ratio.
   (iii)Distance moved by the effort.

$E = 300N, \quad D_L = 2m, \quad L = 900N, \quad D_E = ?, \quad \eta = 75\%$

(i) Mechanical advantage

$$M.A = \frac{L}{E}$$
$$M.A = \frac{900}{300}$$
$$M.A = 3$$

(ii) Velocity ratio

$$\eta = \frac{M.A}{V.R} \times 100\%$$
$$75\% = \frac{3}{V.R} \times 100\%$$
$$75\% V.R = 300\%$$
$$V.R = \frac{300\%}{75\%}$$
$$V.R = \frac{300\%}{75\%}$$
$$V.R = 4$$

(iii) Distance moved by effort

$$V.R = \frac{D_E}{D_L}$$
$$4 = \frac{D_E}{2}$$
$$D_E = 4 \times 2$$
$$D_E = 8m$$

EXERCISE:

1. An effort of 100N moves through 12cm while moving a Load of 400N through 2cm. Find;
   i) the mechanical advantage.
   ii) the velocity ratio
   iii) the efficiency of the machine.
2. A water pump raises 2000kg of water through a vertical height of 22m. If the efficiency of the water pump is 80%, calculate the work input.
3. A simple machine raises a load of 60N through a distance of 2m by an effort of 20N which moves through a distance of 8m. Calculate the machine's efficiency.
4. A load of 100N is raised through 6m when an effort of 40N moves through a distance of 24m. Calculate the;
   i) mechanical advantage.
   ii) velocity ratio.
   iii) energy wasted by the machine.
   iv) efficiency of the machine.
5. A simple machine raises a load of 300kg through 0.5m when an effort of 150N is applied through a distance of 12.5m. Calculate the;
   i) work input into machine.
   ii) work output by the machine.
   iii) efficiency of the machine.

Examples of simple machines include;

- Levers
- Pulleys
- Inclined planes
- Hydraulic machines
- Wheel and axle
- Gears
- Screws
- wedges

LEVERS

A lever is a rigid body is free to turn about a fixed point.
The fixed point at which the lever turns is called the **pivot** or **fulcrum**.

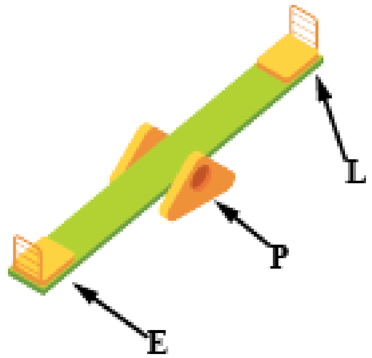
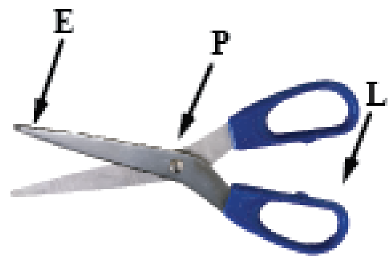
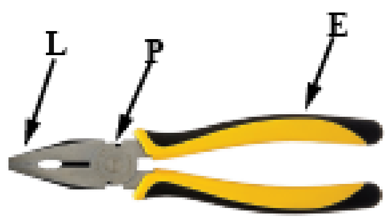
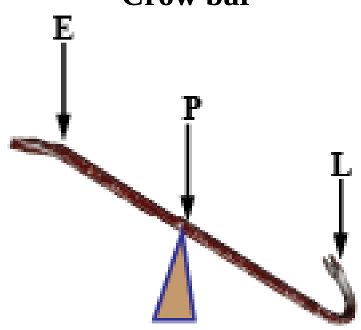
Classes of levers:
There are three classes of levers namely;

- First class lever.
- Second class lever.
- Third class lever.

FIRST CLASS LEVERS
This is the type of lever where the pivot is between the load and the effort (LPE).

Examples include;

- Crow bar
- Beam balance
- Pair of scissors
- Pair of pliers
- See saw
- Claw hammer
- Shears
- Secateurs

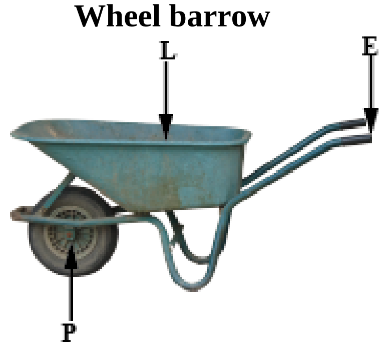
See saw	Pair of scissors	Claw hammer
		
Pair of pliers	**Crow bar**	
		

SECOND CLASS LEVERS

This is the type of lever where the load is between the pivot and the effort (PLE).

Examples include;

- Wheel barrow
- Nut cracker
- Bottle opener
- Office punching machine

Wheel barrow	Nut cracker	Bottle opener
		

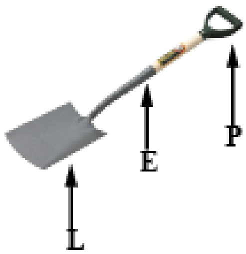
THIRD CLASS LEVERS

This is the type of lever where the effort is between the load and the pivot (LEP).

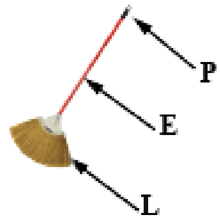
Examples include;

- Spade
- Pair of tongs
- Tweezers
- Broom
- Fishing rod
- Stapling machine

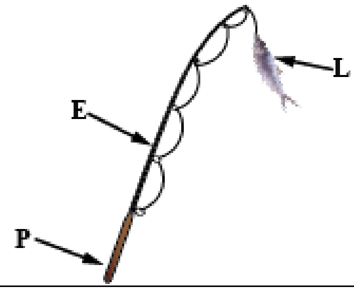
Spade



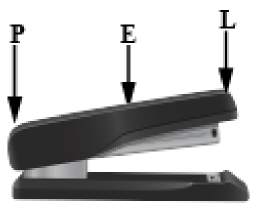
Broom



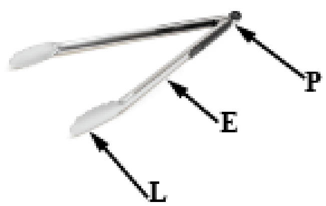
Fishing rod



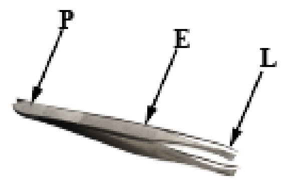
Stapling machine



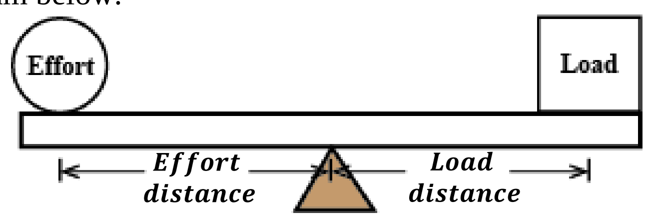
Forceps



Tweezers



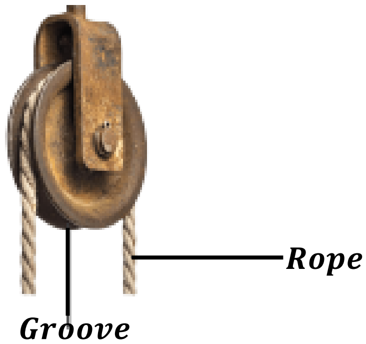
NOTE:
Consider the diagram below.



- The operation of a lever depends on the principle of moments.
- The efficiency of a lever can be increased by the effort distance (distance of the effort from the turning point).

PULLEYS

A pulley is a wheel with a grooved rim on which a rope passes.



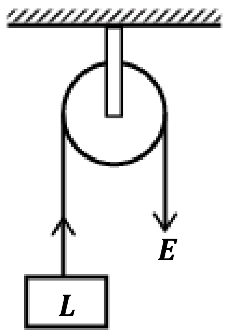
Types of pulley systems:

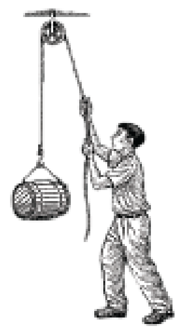
There are three types of pulleys namely;

- Single fixed pulley.
- Single movable pulley.
- Block and tackle pulley.

SINGLE FIXED PULLEY:

This is a type of pulley system in which the pulley is fixed on the rigid support.
In a single fixed pulley, the load is tied to one end and the effort applied to another end of the rope.
As the rope is pulled downwards, the load is raised upwards.
Therefore, a single fixed pulley eases work by changing the direction of the application of the effort.





Assuming, there is no friction in the groove and the rope is weightless;

At equilibrium;

$$\boldsymbol{Upward\ forces = Downward\ forces}$$
$$\boldsymbol{L = E}$$

$\boldsymbol{Therefore,}$ $\quad \boldsymbol{M.A = \frac{L}{E} = \frac{E}{E} = 1}$

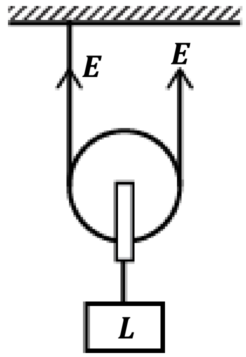
In real practice, the effort is always greater than the load because it is used overcome friction in the groove and also used to lift the weight of the groove. Therefore, mechanical advantage is always less than 1.

However, the distance moved by the effort is always equal to the distance moved by the load i.e.
$\boldsymbol{D_E = D_L}$

$\boldsymbol{Therefore,}$ $\quad \boldsymbol{V.R = \frac{D_E}{D_L} = \frac{D_L}{D_L} = 1}$

SINGLE MOVABLE PULLEY:

This is the type of pulley system in which the pulley moves along with the rope.
One end of the rope is fixed to a rigid support and the effort is applied on the other end.
The advantage of this pulley system is that less effort is required to lift the load thus raising it easily.



Assuming, there is no friction in the groove and the rope is weightless;
At equilibrium;

$$Upward\ forces = Downward\ forces$$
$$L = E + E$$
$$L = 2E$$

$$Therefore, \quad M.A = \frac{L}{E} = \frac{2E}{E} = 2$$

In real practice, the effort is always greater than the load because it is used overcome friction in the groove and also used to lift the weight of the groove. Therefore, mechanical advantage is always less than 2.

However, the distance moved by the effort is always twice the distance moved by the load i.e.
$D_E = 2D_L$

$$Therefore, \quad V.R = \frac{D_E}{D_L} = \frac{2D_L}{D_L} = 2$$

BLOCK AND TACKLE PULLEY SYSTEM:

This is the type of pulley system where two or more pulleys are combined to form a machine with high velocity ratio and high mechanical advantage.
It consists of two blocks namely;

- Fixed block.
- Movable block.

These blocks are joined by a single rope called the "**tackle**".

NOTE:

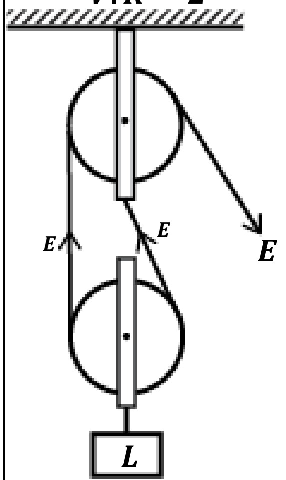
- Velocity ratio is equal to the number of strings supporting the movable block.
- Velocity ratio is also equal to the number of pulleys on the system.
- The effort applied is equal to the tension in each string supporting the movable block.
- For an odd number of pulleys in the system, the fixed block should have one more pulley than the movable block.
- For an even number of pulleys in the system, the fixed and the movable blocks should have the same number of pulleys.

PASSING THE STRINGS (ROPES):

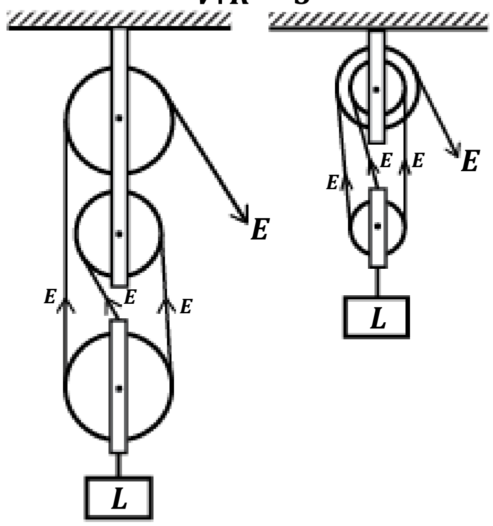
- ❖ If the number of pulleys is odd (velocity ratio is odd), the string must be tied and start from the movable block.
- ❖ If the number of pulleys is even (velocity ratio is even), the string must be tied and start from the fixed block.

Examples of block and tackle pulley systems include;

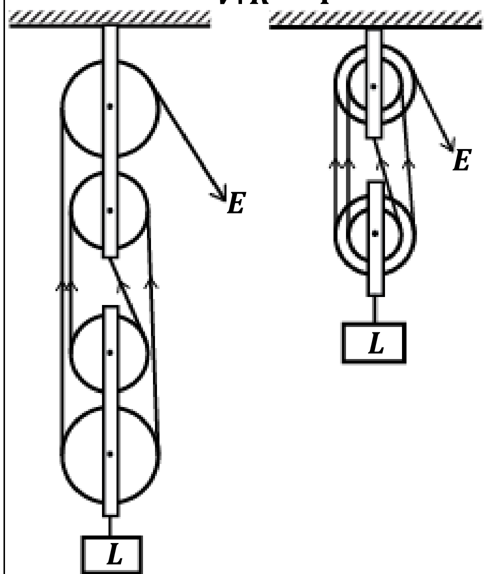
$V.R = 2$



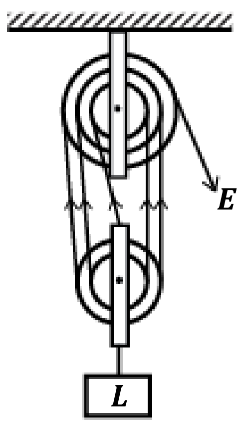
$V.R = 3$



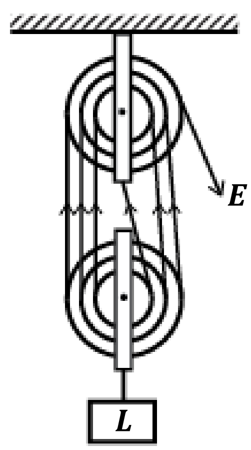
$V.R = 4$



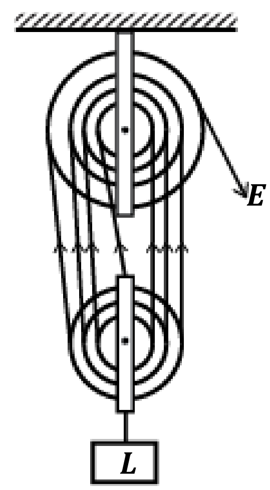
$V.R = 5$



$V.R = 6$

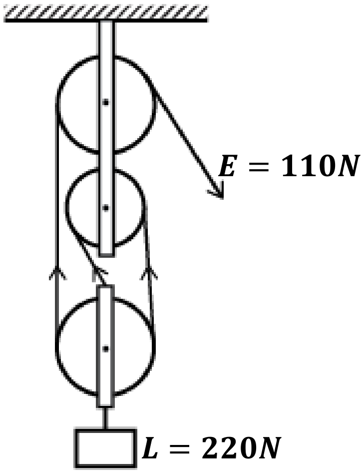


$V.R = 7$



Examples:

1. A block and tackle pulley system shown in the figure below is used to lift a load of 220N when an effort of 110N is applied.
   (i) State the velocity ratio of the system.
   (ii) Calculate the mechanical advantage of the system.
   (iii) Calculate the efficiency of the pulley system.



(i)

$\boldsymbol{Velocity\ ratio = 3}$

(ii) Mechanical advantage

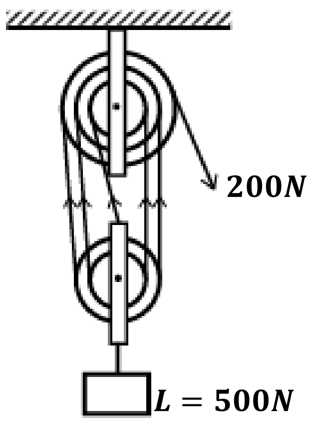
$$M.A = \frac{L}{E}$$
$$M.A = \frac{220}{110}$$
$$M.A = 2$$

(iii) Efficiency

$$\eta = \frac{M.A}{V.R} \times 100\%$$
$$\eta = \frac{2}{3} \times 100\%$$
$$\eta = 66.7\%$$

2. A pulley system of velocity ratio 5 is used to lift a load of 500N. The effort needed is found to be 200N.
   a) Draw the arrangement of the above system.
   b) Calculate the efficiency of the system.

(a)



(b)

$V.R = 5$

Mechanical advantage

$$M.A = \frac{L}{E}$$
$$M.A = \frac{500}{200}$$
$$M.A = 2.5$$

Efficiency

$$\eta = \frac{M.A}{V.R} \times 100\%$$
$$\eta = \frac{2.5}{5} \times 100\%$$
$$\eta = 0.5 \times 100\%$$
$$\eta = 50\%$$

3. A man uses a block and tackle pulley system to raise a load of 720N through a distance of 10m using an effort of 200N. If the pulley system has a velocity ratio of 5, find the;
   a) Mechanical advantage.

$$M.A = \frac{L}{E}$$
$$M.A = \frac{720}{200}$$
$$M.A = 3.6$$

   b) Efficiency.

$$V.R = 5$$
$$\eta = \frac{M.A}{V.R} \times 100\%$$
$$\eta = \frac{3.6}{5} \times 100\%$$
$$\eta = 0.72 \times 100\%$$
$$\eta = 72\%$$

   c) Distance moved by effort.

$$D_L = 10m,\ D_E = ?$$
$$V.R = \frac{D_E}{D_L}$$
$$5 = \frac{D_E}{10}$$
$$D_E = 5 \times 10$$
$$D_E = 50m$$

d) Work input.

$D_E = 50m, \quad E = 200N$

$W.I = E \times D_E$

$W.I = 200 \times 50$

$W.I = 10{,}000J$

e) Work output.

$D_L = 10m, \quad L = 720N$

$W.O = L \times D_L$

$W.O = 720 \times 10$

$W.O = 7200J$

f) Energy wasted

$Energy\ wasted = W.I - W.O$

$E.W = 10{,}000 - 7{,}200$

$E.W = 2{,}800J$

4. A block and tackle pulley system with a velocity ratio of 5 and 60% efficient is used to lift a load of 60kg through a vertical height of 2m. Calculate the effort that must be applied on the system.

$V.R = 5, \quad \eta = 60\%, \quad L = 60 \times 10 = 600N, \quad D_L = 2m$

$$\eta = \frac{M.A}{V.R} \times 100\%$$

$$60\% = \frac{M.A}{5} \times 100\%$$

$$60\% \times 5 = 100\% M.A$$

$$M.A = \frac{300\%}{100\%}$$

$$M.A = 3$$

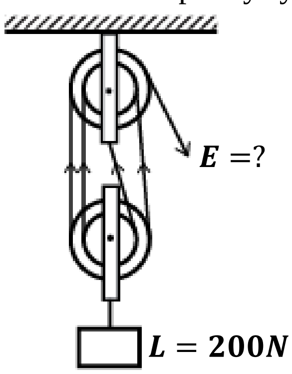
$$M.A = \frac{L}{E}$$

$$3 = \frac{600}{E}$$

$$E = \frac{600}{3}$$

$$E = 200N$$

5. Below is a frictionless pulley system of velocity ratio 4.



If the weight of the pulley system is 4N, calculate;
(i) Effort required to raise the load.
(ii) Mechanical advantage of the system.
(iii)Efficiency of the system.

$Upward\ forces = Downward\ forces$

$Sum\ of\ Tension\ (Efforts)\ in\ the\ strings = Load + Weight\ of\ pulleys + Friction$

$E + E + E + E = 200 + 4 + 0$

$$4E = 204$$

$$E = \frac{204}{4}$$

$$E = 51N$$

<u>Mechanical advantage</u>

$$M.A = \frac{L}{E}$$

$$M.A = \frac{200}{51}$$

$$M.A = 3.92$$

<u>Efficiency</u>

$V.R = 4$

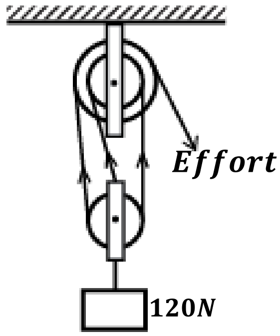
$$\eta = \frac{M.A}{V.R} \times 100\%$$

$$\eta = \frac{3.92}{4} \times 100\%$$

$$\eta = 0.98 \times 100\%$$

$$\eta = 98\%$$

6. The pulley system below has a mass of 0.6kg



Calculate;
(i) Effort required to raise the load.
(ii) Mechanical advantage.
(iii) Efficiency of the pulley system.

(i) Effort

$$Weight\ of\ pulley = mg$$
$$W = 0.6 \times 10 = 6N$$

$$Upward\ forces = Downward\ forces$$
$$E + E + E = L + W$$
$$3E = 120 + 6$$
$$E = \frac{126}{3}$$
$$E = 42N$$

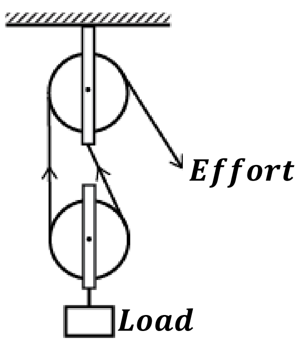
(ii) Mechanical advantage

$$M.A = \frac{L}{E}$$
$$M.A = \frac{120}{42}$$
$$M.A = 2.86$$

(iii) Efficiency

$$V.R = 3$$
$$\eta = \frac{M.A}{V.R} \times 100\%$$
$$\eta = \frac{2.86}{3} \times 100\%$$
$$\eta = 95.3\%$$

7.



The figure besides shows a load of 10N being raised slowly by the aid of a simple frictionless pulley system.
(i) State the velocity ratio of the system.
(ii) Calculate the effort required to raise the load if the mass of each pulley is 0.2kg.
(iii) If the load is raised through a distance of 5m, calculate the efficiency of the pulley system.

(i) $Velocity\ ratio = 2$

(ii) Effort

$$Weight\ of\ movable\ pulley = mg$$
$$W = 0.2 \times 10 = 2N$$

$$Upward\ forces = Downward\ forces$$
$$E + E = L + W$$
$$2E = 10 + 2$$
$$E = \frac{12}{2}$$
$$E = 6N$$

Mechanical advantage

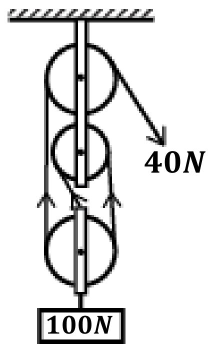
$$M.A = \frac{L}{E}$$
$$M.A = \frac{10}{6}$$
$$M.A = 1.667$$

(iii) Efficiency

$$V.R = 2$$
$$\eta = \frac{1.667}{2} \times 100\%$$
$$\eta = 0.8335 \times 100\%$$
$$\eta = 83.35\%$$

EXERCISE:

1. The effort required to raise a load of 100N is 40N as shown below.



Calculate;
a) Mechanical advantage.
b) Efficiency.
c) Work done on the load if it is raised through a distance of 6m.

2. A block and tackle pulley system is used to lift a mass of 2000kg. If this machine has a velocity ratio of 5 and an efficiency of 80%,
   (i) Sketch a possible arrangement of the pulleys.
   (ii) Calculate the mechanical advantage of the system.
   (iii) Determine the effort applied.

3. An effort of 125N is used to lift a load of 500N through a height of 2.5m using a pulley system. If the distance moved by the effort is 15m, calculate;
   a) the work done on the load.
   b) the work done by the effort.
   c) Efficiency of the pulley system.

4. An effort of 50N is required to raise a load of 200N using a pulley system of velocity ratio 5.
   a) Draw a diagram to show the pulley system.
   b) Find the efficiency of the system.
   c) Calculate the work wasted when the load is raised through 120cm.
   d) Give two reasons why the efficiency of the above pulley is less than 100%.

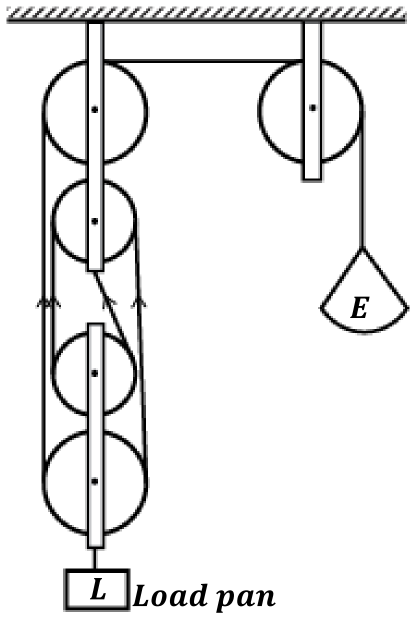
5. A pulley system of velocity ratio 3 supports a load of 20N. given that the tension in each string is 8N, calculate;
   (i) The effort required to raise the load.
   (ii) The mechanical advantage.
   (iii) The efficiency of the pulley system.
   (iv) The distance moved by the effort if the load moves through a distance of 2m.
   (v) The weight of the lower pulley.

6. Draw a diagram showing a single string pulley system of velocity ratio 6. Given that the weight of the lower block and the pulleys is 10N, calculate the efficiency of the pulley system if an effort of 1500N is required to raise a load of 4990N.

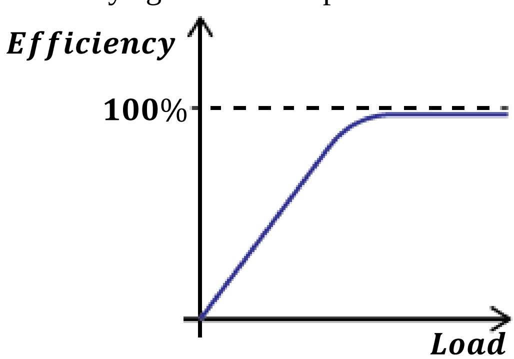
7. A block and tackle pulley system has a velocity ratio of 4. If its efficiency is 75%, find;
   a) Mechanical advantage.
   b) Load that can be lifted with an effort of 500N.
   c) Work done if the load is raised through a vertical distance of 4.0m.

EXPERIMENT TO SHOW THE VARIATION OF MECHANICAL ADVANTAGE OR EFFICIENCY OF PULLEY SYSTEM WITH THE LOAD

(Experiment to determine efficiency of a pulley system)



- A known load (L) is placed on the load pan.
- Known weights are added on the effort pan until the load just begins to rise upwards.
- The total weight (E) on the effort pan is noted and recorded.
- The experiment is repeated with different loads.
- The results are put in a suitable table including values of mechanical advantage and efficiency.
- A graph of efficiency against load is plotted.



Explanation of the graph:
As the load increases, the efficiency of the pulley system also increases. This is because;

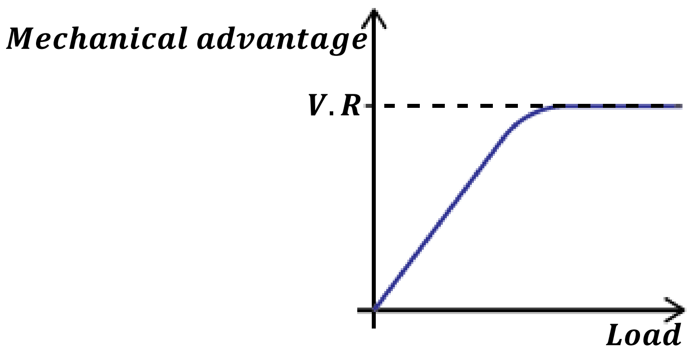
- *When the load is small, a large effort is used to overcome friction force between moving parts and lift the weight of the movable block. This leads to a small mechanical advantage and small efficiency for a small load.*
- *When the load is increased, the friction force and the weight of the movable become very small. Therefore, a large portion of the effort is used to lift the load while a small portion of the effort overcomes friction and lifts the weight of the movable block. This leads to a large mechanical advantage and large efficiency for a small load.*

NOTE:

- The velocity ratio of the above pulley system is 4 but not 5 since we consider the arrangement of pulleys where the load is attached.
- The of table of results is as shown below.

$L(N)$	E(N)	$\frac{L}{E}\ (M.A)$	$\frac{M.A}{V.R} \times 100\%\ (Efficiency)$

- **If the variation of mechanical advantage with load is required, then a graph of mechanical advantage is plotted as shown below.**



APPLICATIONS (USES) OF PULLEYS

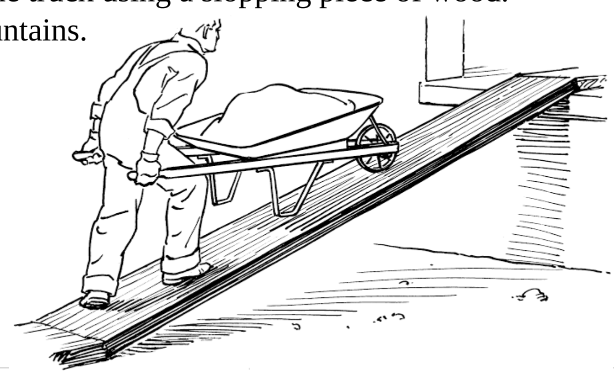
- They are used at construction sites to lift heavy building materials from the ground.
- They are used in raising (hoisting) flags.
- They are used in lifts and elevators.
- They are used in cranes for loading and offloading ships at ports.
- They are used in fetching water from underground wells.
- They are used in drawing stage curtains in theatres.

INCLINED PLANES

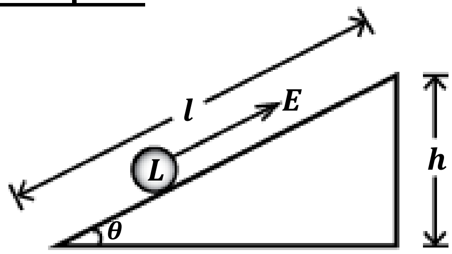
An inclined plane is a sloping surface or plane inclined at angle to the ground.
An inclined plane allows a load to be raised using a small effort than it were to be lifted vertically upwards.

Examples of inclined planes;

- Stair cases.
- Raising cows up to the truck using a slopping piece of wood.
- Sloping roads in mountains.



Structure of an inclined plane



$The\ distance\ moved\ by\ the\ effort\ (D_E) = length\ of\ the\ plane (l)$
$The\ distance\ moved\ by\ the\ load\ (D_L) = height\ of\ the\ plane (h)$

Velocity ratio of an inclined plane:

$$V.R = \frac{Distance\ moved\ by\ the\ effort\ (D_E)}{Distance\ moved\ by\ the\ load\ (D_L)}$$

$$V.R = \frac{length\ of\ the\ plane}{height\ of\ the\ plane}$$

$$\boldsymbol{V.R = \frac{l}{h}}$$

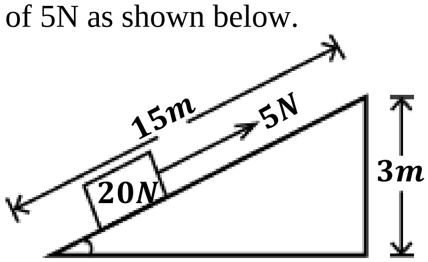
NOTE; Considering the angle of inclination, θ;

$$Since, \quad sin\theta = \frac{h}{l} = \frac{1}{V.R}$$

$$Therfore, \quad \boldsymbol{V.R = \frac{1}{sin\theta}}$$

Examples:

1. A brick of weight 20N is lifted through a height of 3m along a smooth inclined plane of length 15m by applying an effort of 5N as shown below.



Calculate;

i) Mechanical advantage

$\boldsymbol{L = 20N, \quad E = 5N}$

$$\boldsymbol{M.A = \frac{L}{E}}$$

$$\boldsymbol{M.A = \frac{20}{5}}$$

$$\boldsymbol{M.A = 4}$$

ii) Velocity ratio

$\boldsymbol{D_L = 3m, \quad D_E = 15m}$

$$\boldsymbol{V.R = \frac{D_E}{D_L}}$$

$$\boldsymbol{V.R = \frac{15}{3}}$$

$$\boldsymbol{V.R = 5}$$

iii) Efficiency

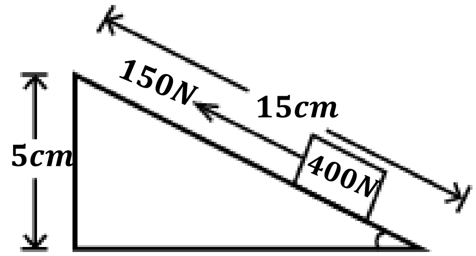
$$\boldsymbol{\eta = \frac{M.A}{V.R} \times 100\%}$$

$$\boldsymbol{\eta = \frac{4}{5} \times 100\%}$$

$$\boldsymbol{\eta = 0.8 \times 100\%}$$

$$\boldsymbol{\eta = 80\%}$$

2. A load of 400N is pulled along an inclined plane as shown below.



Calculate;
i) Work input.
ii) Work output.
iii) Efficiency of the plane.

i) Work input.

$D_E = 15cm = \frac{15}{100} = 0.15m$, $E = 150N$

$W.I = E \times D_E$

$W.I = 150 \times 0.15$

$W.I = 22.5\,J$

ii) Work output.

$D_L = 5cm = \frac{5}{100} = 0.05m$, $L = 400N$

$W.O = L \times D_L$

$W.O = 400 \times 0.05$

$W.O = 20\,J$

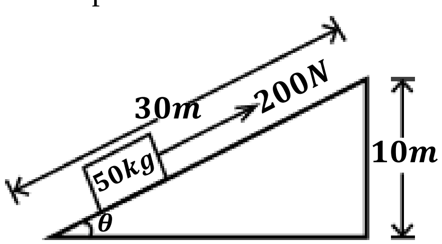
iii) Efficiency

$\eta = \frac{W.O}{W.I} \times 100\%$

$\eta = \frac{20}{22.5} \times 100\%$

$\eta = 88.9\%$

3. An inclined plane shown below was used to lift a load of 50kg using an effort of 200N.



Calculate;
(i) Mechanical advantage.
(ii) Velocity ratio.
(iii) Efficiency.
(iv) Angle of inclination.

i) Mechanical advantage

$L = mg$

$L = 50 \times 10 = 500N$,

$E = 200N$

$M.A = \frac{L}{E}$

$M.A = \frac{500}{200}$

$M.A = 2.5$

ii) Velocity ratio

$D_L = 10m$, $D_E = 30m$

$V.R = \frac{l}{h}$

$V.R = \frac{30}{10}$

$V.R = 3$

iii) Efficiency

$\eta = \frac{M.A}{V.R} \times 100\%$

$\eta = \frac{2.5}{3} \times 100\%$

$\eta = 83.3\%$

iv) Angle of inclination

$\sin\theta = \frac{h}{l} = \frac{10}{30} = \frac{1}{3}$

$\theta = \sin^{-1}\left(\frac{1}{3}\right)$

$\theta = 19.5°$

4. A woman uses an inclined plane to lift a load of 500N through a vertical distance of 4m. the inclined plane makes an angle of 30° to the horizontal. If the efficiency of the inclined plane is 72%, calculate the effort need to raise the load.

$l = ?$, $h = 4m$

$\sin\theta = \frac{h}{l}$

$\sin 30° = \frac{4}{l}$

$l \sin 30° = 4$

$l = \frac{4}{\sin 30°} = 8m$

Then;

$V.R = \frac{l}{h} = \frac{8}{4}$

$V.R = 2$

$\eta = \frac{M.A}{V.R} \times 100\%$

$72\% = \frac{M.A}{2} \times 100\%$

$M.A = 1.44$

$M.A = \frac{L}{E}$

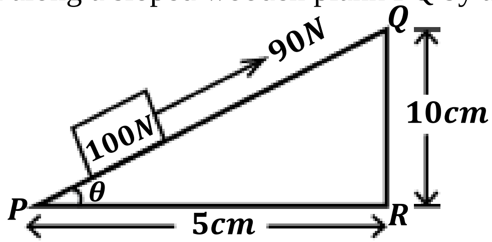
$1.44 = \frac{500}{E}$

$E = 347.2N$

EXERCISE:

1. An effort of 50N is used to move a 300N box along an inclined which rises vertically 1m for every 8m distance along the plane. Find
   i) the velocity ratio.
   ii) the mechanical advantage.
   iii) the efficiency of the inclined plane.

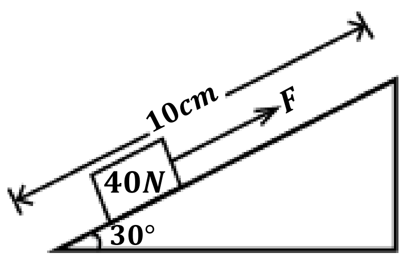
2. A body of 100N is moved along a sloped wooden plank PQ by an effort of 90N as shown below.



   Calculate;
   a) the length of the plane.
   b) angle of inclination.
   c) velocity ratio.
   d) Mechanical advantage.
   e) Efficiency of the plane.

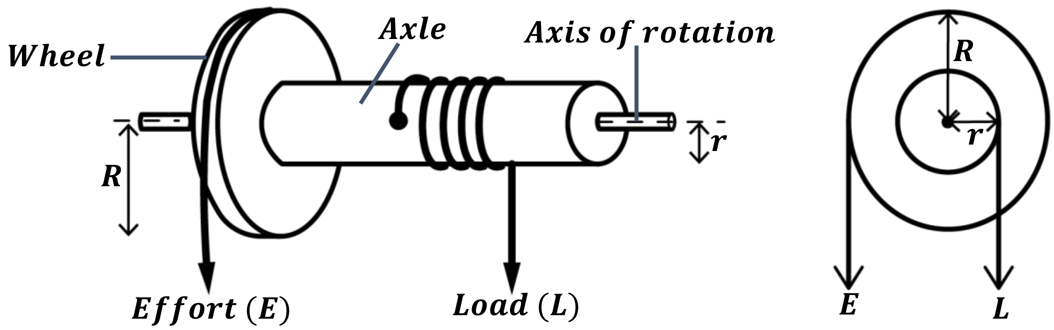
3. A trolley of weight 10N is pulled from the bottom to the top of the inclined plane by a steady force of 2N. If the height and the distance moved by the force are 2m and 20m respectively, calculate;
   a) Mechanical advantage.
   b) Work done by the effort.
   c) Work done on the load.
   d) Efficiency of the inclined plane.

4. A load of 40N is pulled steadily along an 80% efficient inclined plane by a force, F as shown below. Find the
   i) Velocity ratio of the system
   ii) Mechanical advantage.
   iii) Force, F.



WHEEL AND AXLE

It consists of a wheel of large radius attached to an axle of small radius. The wheel and axle have a common axis of rotation.



The effort ($\boldsymbol{E}$) is applied to one end of the rope passing over the wheel of radius, $\boldsymbol{R}$ while the load is applied at the other end of the rope passing over the axle of radius, $\boldsymbol{r}$.

The wheel and the axle are circular therefore, for one complete turn;

- The effort moves through a distance equal to the circumference of the wheel ($\boldsymbol{2\pi R}$).
- The load moves through a distance equal to the circumference of the axle ($\boldsymbol{2\pi r}$).

Velocity ratio of a wheel and axle:

$$V.R = \frac{Distance\ moved\ by\ the\ effort\ (D_E)}{Distance\ moved\ by\ the\ load\ (D_L)}$$

$$V.R = \frac{2\pi R}{2\pi r}$$

$$\boldsymbol{V.R = \frac{R}{r}}$$

Therefore, velocity ratio of a wheel and axle is given by;

$$\boldsymbol{V.R = \frac{Radius\ of\ the\ wheel\ (R)}{Radius\ of\ the\ axle\ (r)}}$$

Examples:

1. A machine consisting of a wheel of radius 50cm and axle of radius 10cm is used to lift a load of 400N with an effort of 100N. Calculate the efficiency of the machine.

Mechanical advantage

$$L = 400N, \quad E = 100N$$
$$M.A = \frac{L}{E}$$
$$M.A = \frac{400}{100}$$
$$M.A = 4$$

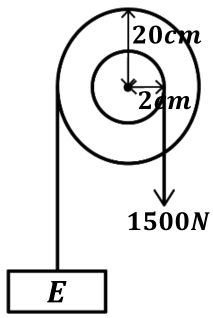
Velocity ratio

$$R = 50cm, \quad r = 10cm$$
$$V.R = \frac{R}{r}$$
$$V.R = \frac{50}{10}$$
$$V.R = 5$$

Efficiency

$$\eta = \frac{M.A}{V.R} \times 100\%$$
$$\eta = \frac{4}{5} \times 100\%$$
$$\eta = 0.8 \times 100\%$$
$$\eta = 80\%$$

2. The efficiency of the machine below is 75%.



Calculate;
i) its velocity ratio.
ii) its mechanical advantage.
iii) the effort applied.

i) Velocity ratio

$$R = 20cm, \quad r = 2cm$$
$$V.R = \frac{R}{r}$$
$$V.R = \frac{20}{2}$$
$$V.R = 10$$

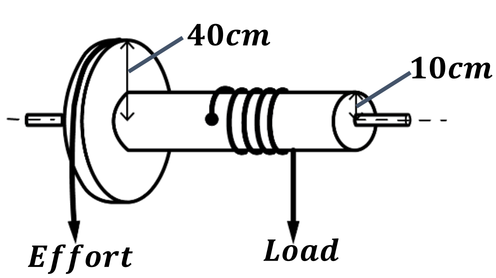
ii) Mechanical advantage

$$\eta = \frac{M.A}{V.R} \times 100\%$$
$$75\% = \frac{M.A}{10} \times 100\%$$
$$75\% \times 10 = 100\% M.A$$
$$M.A = \frac{750\%}{100\%}$$
$$M.A = 7.5$$

iii) Effort

$$M.A = \frac{L}{E}$$
$$7.5 = \frac{1500}{E}$$
$$E = \frac{1500}{7.5}$$
$$E = 200N$$

3.



The figure besides shows a wheel and axle. When an effort of 300N is applied, a load of 900N is raised. Calculate;
a) Work output.
b) Work input.
c) Efficiency.

$$R = 40cm = \frac{40}{100} = 0.4m$$
$$r = 10cm = \frac{10}{100} = 0.1m$$

$\boldsymbol{Effort\ distance,}\quad D_E = 2\pi R = 2 \times \pi \times 0.4 = \mathbf{0.8\pi\ m}$

$\boldsymbol{Load\ distance,}\quad D_L = 2\pi r = 2 \times \pi \times 0.1 = \mathbf{0.2\pi\ m}$

i) Work output

$L = 900N$

$\boldsymbol{W.O = L \times D_L}$

$\boldsymbol{W.O = 900 \times 0.2\pi}$

$\boldsymbol{W.O = 180\pi\ J}$

ii) Work input

$E = 900N$

$\boldsymbol{W.I = E \times D_E}$

$\boldsymbol{W.I = 300 \times 0.8\pi}$

$\boldsymbol{W.I = 240\pi\ J}$

iii) Efficiency

$$\boldsymbol{\eta = \frac{Work\ output}{Work\ input} \times 100\%}$$

$$\boldsymbol{\eta = \frac{180\pi}{240\pi} \times 100\%}$$

$$\boldsymbol{\eta = 0.75 \times 100\%}$$

$$\boldsymbol{\eta = 75\%}$$

4. In a wheel and axle machine, an effort of 10N raises a load of 30N. The radius of the wheel is 150mm and the radius of the axle is 30mm. Find the efficiency of the machine.

Mechanical advantage

$\boldsymbol{L = 30N, \quad E = 10N}$

$$\boldsymbol{M.A = \frac{L}{E}}$$

$$\boldsymbol{M.A = \frac{30}{10}}$$

$$\boldsymbol{M.A = 3}$$

Velocity ratio

$\boldsymbol{R = 150mm, \quad r = 30mm}$

$$\boldsymbol{V.R = \frac{R}{r}}$$

$$\boldsymbol{V.R = \frac{150mm}{30mm}}$$

$$\boldsymbol{V.R = 5}$$

Efficiency

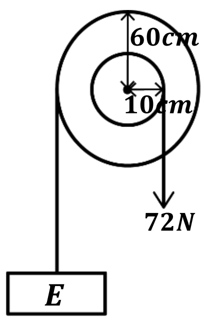
$$\boldsymbol{\eta = \frac{M.A}{V.R} \times 100\%}$$

$$\boldsymbol{\eta = \frac{3}{5} \times 100\%}$$

$$\boldsymbol{\eta = 0.6 \times 100\%}$$

$$\boldsymbol{\eta = 60\%}$$

5. The diagram below shows a 75% simple machine used to raise a load of 72N.



a) Name the type of machine above.

$\boldsymbol{Wheel\ and\ axle}$

b) Determine the effort required to raise the load.

Velocity ratio

$\boldsymbol{R = 60cm, \quad r = 10cm}$

$$\boldsymbol{V.R = \frac{R}{r}}$$

$$\boldsymbol{V.R = \frac{60}{10}}$$

$$\boldsymbol{V.R = 6}$$

Mechanical advantage

$$\boldsymbol{\eta = \frac{M.A}{V.R} \times 100\%}$$

$$\boldsymbol{75\% = \frac{M.A}{6} \times 100\%}$$

$$\boldsymbol{75\% \times 6 = 100\% M.A}$$

$$\boldsymbol{M.A = \frac{450\%}{100\%}}$$

$$\boldsymbol{M.A = 4.5}$$

Effort

$$\boldsymbol{M.A = \frac{L}{E}}$$

$$\boldsymbol{4.5 = \frac{72}{E}}$$

$$\boldsymbol{E = \frac{72}{4.5}}$$

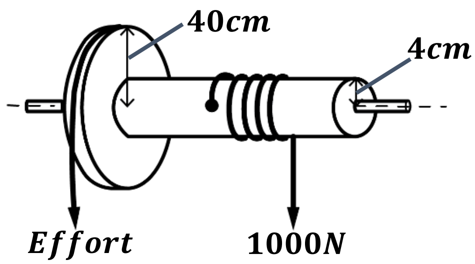
$$\boldsymbol{E = 16N}$$

Applications of wheel and axle in daily life

- It is applied in screw drivers.
- It is applied in steering wheels in cars.
- It is applied in a wind las to draw water from wells.

EXERCISE

1. A machine consists of a wheel of 40cm and an axle of radius 10cm. If an effort of 20N raises a load of 60N, find the efficiency of the machine.
2. The system below is a wheel and axle of radii 40cm and 4cm respectively.



Assuming that the efficiency of the system is 50%, calculate;
a) Effort required.
b) Work output.
c) Work input.
d) Energy wasted.

3. A wheel and axle machine is constructed from a wheel of diameter 20cm and mounted on an axle of diameter 4cm.
   a) Calculate;
      i) Velocity ratio of the machine.
      ii) Mechanical advantage of the machine if its 100%.
   b) Explain why the actual mechanical advantage of this machine is likely to be less than the value obtained above.
4. A common windlass is used to raise a load of 480N by an application of an effort 200N at right angles to the handle. If the handle has a radius of 33cm from the axis and the radius of the axle is 11cm, calculate;
   a) Velocity ratio.
   b) Efficiency of the windlass.

GEARS

A gear is a device consisting of a set of toothed wheels that control the movement (speed) of a machine.

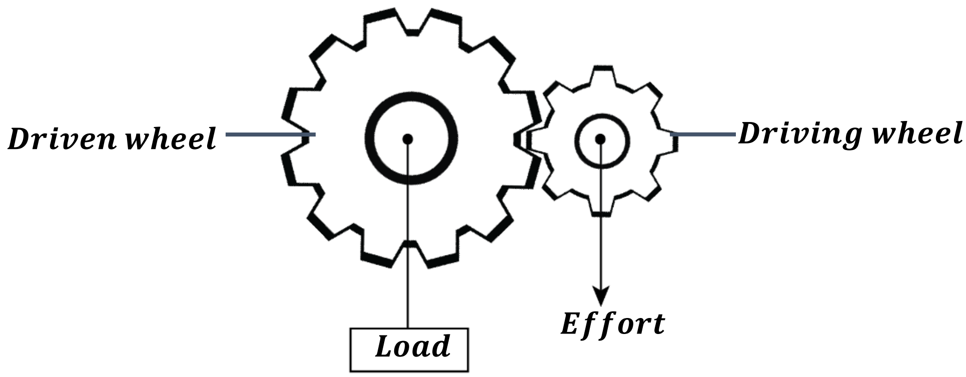
Applications of gears in daily life

They are applied in;

- Clocks
- Motors
- Bicycles
- Watches
- Motor vehicles

In gears;

- The effort is applied to the shaft of the small gear (wheel) called a **driving wheel**.
- The load is applied to the shaft of the large gear (wheel) called a **driven wheel**.



NOTE:

- The more the number of teeth on the gear, the less the speed of rotation of the gear and the less the number of teeth on the gear, the higher the speed of rotation of the gear.
- Therefore, the fastest gear is the driving wheel with the smallest number of teeth.

Velocity ratio of a gear system:

Velocity ratio of a gear system may be given by the following formulae;

$$V.R = \frac{Number\ of\ teeth\ of\ driven\ wheel}{Number\ of\ teeth\ of\ driving\ wheel}$$

$$V.R = \frac{Speed\ of\ rotation\ of\ driven\ wheel}{Speed\ of\ rotation\ of\ driving\ wheel}$$

$$V.R = \frac{Number\ of\ revolutions\ of\ driving\ wheel}{Number\ of\ revolutions\ of\ driven\ wheel}$$

Examples:

1. A driving wheel of 25 teeth interlocks with another wheel of 100 teeth. The gear system has an efficiency of 85%.
   Calculate;
   a) Velocity ratio.
   b) Mechanical advantage of the system.

a)

$$No.of\ teeth\ on\ driving\ wheel = 25$$
$$No.of\ teeth\ on\ driven\ wheel = 100$$

$$V.R = \frac{Number\ of\ teeth\ of\ driven\ wheel}{Number\ of\ teeth\ of\ driving\ wheel}$$

$$V.R = \frac{100}{25}$$

$$V.R = 4$$

b)

$$\eta = \frac{M.A}{V.R} \times 100\%$$

$$85\% = \frac{M.A}{4} \times 100\%$$

$$85\% \times 4 = 100\% M.A$$

$$M.A = \frac{340\%}{100\%}$$

$$M.A = 3.4$$

2. A bicycle has 120 teeth in the driven gears and 40 teeth in the driving gears. Calculate;
   a) Velocity ratio.
   b) Mechanical advantage if the bicycle is 80% efficient.

a) Velocity ratio

$$No. of\ teeth\ on\ driving\ wheel = 40$$
$$No. of\ teeth\ on\ driven\ wheel = 120$$

$$\boldsymbol{V.R = \frac{Number\ of\ teeth\ of\ driven\ wheel}{Number\ of\ teeth\ of\ driving\ wheel}}$$
$$\boldsymbol{V.R = \frac{120}{40}}$$
$$\boldsymbol{V.R = 3}$$

b) Mechanical advantage

$$\boldsymbol{\eta = \frac{M.A}{V.R} \times 100\%}$$
$$\boldsymbol{80\% = \frac{M.A}{3} \times 100\%}$$
$$\boldsymbol{80\% \times 3 = 100\% M.A}$$
$$\boldsymbol{M.A = \frac{240\%}{100\%}}$$
$$\boldsymbol{M.A = 2.4}$$

3. In a gear system, the driven wheel has 40 teeth and the driving wheel has 10 teeth. The system is used to carry a load of 300N when an effort of 100N is applied. Determine;
   i) Velocity ratio.
   ii) Mechanical advantage.
   iii) Efficiency.

i) Velocity ratio

$$No. of\ teeth\ on\ driving\ wheel = 10$$
$$No. of\ teeth\ on\ driven\ wheel = 40$$

$$V.R = \frac{Number\ of\ teeth\ of\ driven\ wheel}{Number\ of\ teeth\ of\ driving\ wheel}$$
$$\boldsymbol{V.R = \frac{40}{10}}$$
$$\boldsymbol{V.R = 4}$$

ii) M.A

$$\boldsymbol{M.A = \frac{L}{E}}$$
$$\boldsymbol{M.A = \frac{300}{100}}$$
$$\boldsymbol{M.A = 3}$$

iii) Efficiency

$$\boldsymbol{\eta = \frac{M.A}{V.R} \times 100\%}$$
$$\boldsymbol{\eta = \frac{3}{4} \times 100\%}$$
$$\boldsymbol{\eta = 0.75 \times 100\%}$$
$$\boldsymbol{\eta = 75\%}$$

4. A certain gear has 60 teeth and drives another gear with 150 teeth. How many revolutions will the driven gear make when the driving gear makes 200 revolutions.

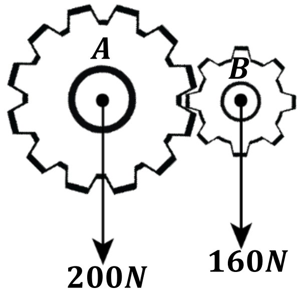
Velocity ratio

$$No. of\ teeth\ on\ driving\ wheel = 60$$
$$No. of\ teeth\ on\ driven\ wheel = 150$$

$$\boldsymbol{V.R = \frac{Number\ of\ teeth\ of\ driven\ wheel}{Number\ of\ teeth\ of\ driving\ wheel}}$$
$$\boldsymbol{V.R = \frac{150}{60}}$$
$$\boldsymbol{V.R = 2.5}$$

$$V.R = \frac{Number\ of\ revolutions\ of\ driving\ wheel}{Number\ of\ revolutions\ of\ driven\ wheel}$$
$$\boldsymbol{2.5 = \frac{200}{R}}$$
$$\boldsymbol{R = \frac{200}{2.5}}$$
$$\boldsymbol{R = 80\ revolutions}$$

5. The figure below shows two gear wheels.



a) How should gears A and B engage each other so that a low mechanical advantage is obtained.
A should be the driven gear since it has more teeth and B should be the driving gear since it has less teeth.

b) Calculate the efficiency of the gear system.

<u>Velocity ratio</u>

$$No.of\ teeth\ on\ driving\ wheel = 8$$
$$No.of\ teeth\ on\ driven\ wheel = 12$$

$$V.R = \frac{Number\ of\ teeth\ of\ driven\ wheel}{Number\ of\ teeth\ of\ driving\ wheel}$$

$$\boldsymbol{V.R = \frac{12}{8}}$$
$$\boldsymbol{V.R = 1.5}$$

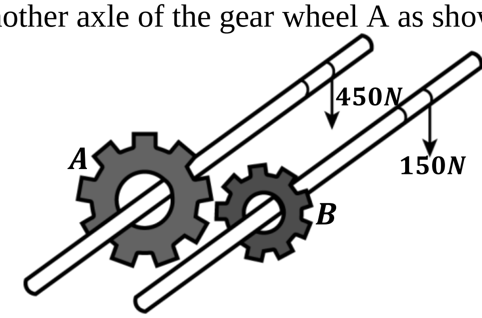
<u>M.A</u>

$$\boldsymbol{M.A = \frac{L}{E}}$$
$$\boldsymbol{M.A = \frac{200}{160}}$$
$$\boldsymbol{M.A = 1.25}$$

<u>Efficiency</u>

$$\boldsymbol{\eta = \frac{M.A}{V.R} \times 100\%}$$
$$\boldsymbol{\eta = \frac{1.25}{1.5} \times 100\%}$$
$$\boldsymbol{\eta = 83.3\%}$$

6. Two gear wheels A and B with 80 and 20 teeth respectively lock into each other. They are fastened to axles such that a weight of 150N attached to one axle of the gear wheel B raises a load of 450N attached to another axle of the gear wheel A as shown below.



Calculate;

i) Velocity ratio

$$No.of\ teeth\ on\ driving\ wheel\ B = 20$$
$$No.of\ teeth\ on\ driven\ wheel\ A = 80$$

$$V.R = \frac{Number\ of\ teeth\ of\ driven\ wheel}{Number\ of\ teeth\ of\ driving\ wheel}$$

$$\boldsymbol{V.R = \frac{80}{20}}$$
$$\boldsymbol{V.R = 4}$$

ii) M.A

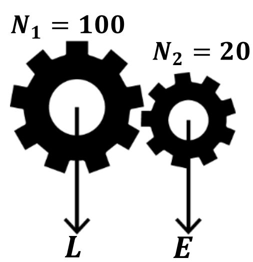
$$\boldsymbol{M.A = \frac{L}{E}}$$
$$\boldsymbol{M.A = \frac{450}{150}}$$
$$\boldsymbol{M.A = 3}$$

iii) Efficiency

$$\boldsymbol{\eta = \frac{M.A}{V.R} \times 100\%}$$
$$\boldsymbol{\eta = \frac{3}{4} \times 100\%}$$
$$\boldsymbol{\eta = 0.75 \times 100\%}$$
$$\boldsymbol{\eta = 75\%}$$

EXERCISE:

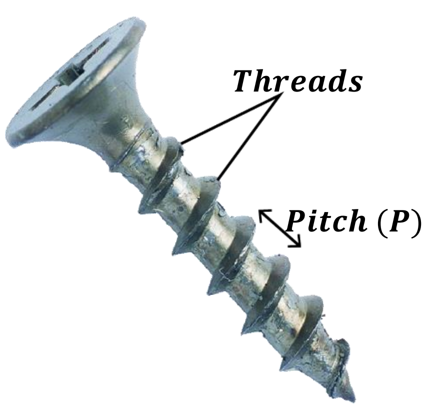
1. A bicycle has a chain wheel with 32 teeth, and the driven wheel has 80 teeth. If the efficiency is 88%, find the;
   i) Velocity ratio.
   ii) Mechanical advantage.
2. A gear with 30 teeth drives another gear with 75 teeth. How many revolutions will the driven gear make when the driving gear makes 100 revolutions.
3. Two gear wheels A and B with 20 teeth and 10 teeth respectively are fastened together such that the weight of 160N is attached to one wheel and rises a load of 400N applied on the other wheel.
   If wheel B drives A, Calculate;
   a) Velocity ratio of the system.
   b) Efficiency.
4. A gear has a driven wheel moving at $20ms^{-1}$ and a driving wheel moving at $4ms^{-1}$. The gear carries a load of 300N and is overcome by an effort of 150N. Calculate the efficiency of the gear system.
5. In the gear system below, N_1 and N_2 are the number of teeth on the system. The gear system has an efficiency of 60%.



   Calculate;
   a) Velocity ratio.
   b) Load that can be raised by an effort of 200N.

SCREWS

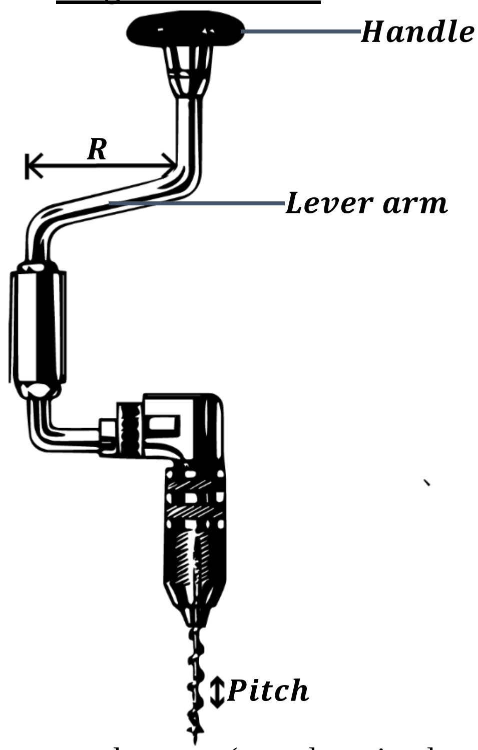
This is a device with thread-like windings on it.
It is used to fix or hold materials together.



Pitch of a screw:
This is the distance between any two successive threads of a screw.

NOTE:
- In order to use a screw, a screw driver or brace or screw jack is used to drive screws in and out of the material.
- An effort is applied on the handles of those devices above to drive the screw (load) in and out of the material.

Diagram of a brace



- When the handle moves through one complete turn (complete circular path), the screw enters or leaves the wood through a distance equal to the pitch of the screw.
- Distance moved by the effort in one complete turn is equal to the circumference of a circle described by the handle.

$$\boldsymbol{Effort\ distance = circumference\ of\ circle\ described\ by\ handle}$$
$$\boldsymbol{D_E = 2\pi R}$$
(Where radius, R of the circle is equal to the length of the lever arm)

- Distance moved by the load (screw) in one complete turn is equal to the pitch of the screw.

$$\boldsymbol{Load\ distance = Pitch\ of\ the\ screw}$$
$$\boldsymbol{D_L = P}$$

Velocity ratio of a screw:

$$V.R = \frac{Distance\ moved\ by\ the\ effort\ (D_E)}{Distance\ moved\ by\ the\ load\ (D_L)}$$

$$V.R = \frac{Circumference\ of\ a\ circle\ described\ by\ handle}{Pitch\ of\ the\ screw}$$

$$\boxed{V.R = \frac{2\pi R}{Pitch}}$$

Examples:

1. In a screw jack, the length of the lever arm is 56cm and a pitch of 4cm. It is used to lift a load. Calculate its velocity ratio.

   $R = 56cm, \quad Pitch = 4cm$

   $$V.R = \frac{2\pi R}{Pitch}$$

   $$V.R = \frac{2 \times \frac{22}{7} \times 56}{4} = \frac{352}{4}$$

   $$V.R = 88$$

2. A screw of pitch 5cm is used to lift a load of 890.8N in a car jack. The lever makes a circle of circumference 10cm and has an efficiency of 85%.
   Calculate;
   a) Velocity ratio of screw.
   b) Mechanical advantage of screw.
   c) Effort applied on the handle.

   a) Velocity ratio

   $C = 56cm, \quad P = 5cm$

   $$V.R = \frac{Circumference}{Pitch}$$

   $$V.R = \frac{10}{5}$$

   $$V.R = 2$$

   b) Mechanical advantage

   $$\eta = \frac{M.A}{V.R} \times 100\%$$

   $$85\% = \frac{M.A}{2} \times 100\%$$

   $$85\% \times 2 = 100\% M.A$$

   $$M.A = \frac{170\%}{100\%}$$

   $$M.A = 1.7$$

   c) Effort

   $$M.A = \frac{L}{E}$$

   $$1.7 = \frac{890.8}{E}$$

   $$E = \frac{890.8}{1.7}$$

   $$E = 524N$$

3. A screw has a pitch of 5mm. If an effort of 30N is rotated through one turn of radius 50cm to lift a load of 750N, find;
   i) Velocity ratio.
   ii) Mechanical advantage.
   iii) Efficiency.

   i) Velocity ratio

   $R = 50cm,$

   $$Pitch = 5mm = \frac{5}{10} = 0.5cm$$

   $$V.R = \frac{2\pi R}{Pitch}$$

   $$V.R = \frac{2 \times 3.14 \times 50}{0.5} = \frac{314}{0.5}$$

   $$V.R = 628$$

   ii) M.A

   $$M.A = \frac{L}{E}$$

   $$M.A = \frac{750}{30}$$

   $$M.A = 25$$

   iii) Efficiency

   $$\eta = \frac{M.A}{V.R} \times 100\%$$

   $$\eta = \frac{25}{628} \times 100\%$$

   $$\eta = 0.0398 \times 100\%$$

   $$\eta = 3.98\%$$

4. A screw with a lever arm of 56cm has two successive threads which are 2.5mm apart. It is used to lift a load of 800N. If its 25% efficient, calculate the mechanical advantage of the screw.

d) Velocity ratio

$$R = 56cm,$$
$$Pitch = 2.5mm = \frac{2.5}{10} = 0.25cm$$
$$V.R = \frac{2\pi R}{Pitch}$$
$$V.R = \frac{2 \times \frac{22}{7} \times 56}{0.25} = \frac{352}{0.25}$$
$$V.R = 1408$$

e) Mechanical advantage

$$\eta = \frac{M.A}{V.R} \times 100\%$$
$$25\% = \frac{M.A}{1408} \times 100\%$$
$$25\% \times 1408 = 100\% M.A$$
$$M.A = \frac{35200\%}{100\%}$$
$$M.A = 352$$

5. The handle of a screw jack is 14cm long. The screw jack is used to drive a screw of pitch 20cm. if an effort of 5N is applied on the jack to move a screw of 15N, calculate
i) Velocity ratio.
ii) Mechanical advantage.
iii) Efficiency.

i) Velocity ratio

$$R = 14cm,$$
$$Pitch = 20cm$$
$$V.R = \frac{2\pi R}{Pitch}$$
$$V.R = \frac{2 \times \frac{22}{7} \times 14}{20} = \frac{88}{20}$$
$$V.R = 4.4$$

ii) M.A

$$M.A = \frac{L}{E}$$
$$M.A = \frac{15}{5}$$
$$M.A = 3$$

iii) Efficiency

$$\eta = \frac{M.A}{V.R} \times 100\%$$
$$\eta = \frac{3}{4.4} \times 100\%$$
$$\eta = 68.2\%$$

6. A screw has 6 successive threads and describes a circular path of diameter 0.28mm when a screw driver is attached to it. Determine the velocity ratio of the machine if the distance between the 6 threads is 0.12mm.

$$R = \frac{diameter}{2} = \frac{0.28}{2} = 0.14mm$$

There are 5 pitches between the 6 threads.

$$5P = 0.12mm$$
$$P = \frac{0.12}{5} = 0.024mm$$

$$V.R = \frac{2\pi R}{Pitch}$$
$$V.R = \frac{2 \times \frac{22}{7} \times 0.14}{0.024} = \frac{0.88}{0.024}$$
$$V.R = 36.67$$

NOTE:
- ***The velocity ratio of the screw is always very large because the length of the handle is very big compared to the pitch of the screw.***
- ***The efficiency is always very low because screws have a very high friction since the threads are very rough. This helps screws to firmly hold materials together.***

EXERCISE:

1. The pitch of a screw jack is 2.5mm. With a lever arm of 56cm long, the jack is used to lift a car of mass 790kg. if the screw jack is 75% efficient, determine;
   a) Velocity ratio
   b) Mechanical advantage.
   c) Effort applied.

2. The pitch of a bolt is 1mm. to tighten the bolt, Ssekwe uses a spanner of a long arm of length 80cm. Calculate the velocity ratio of the spanner.

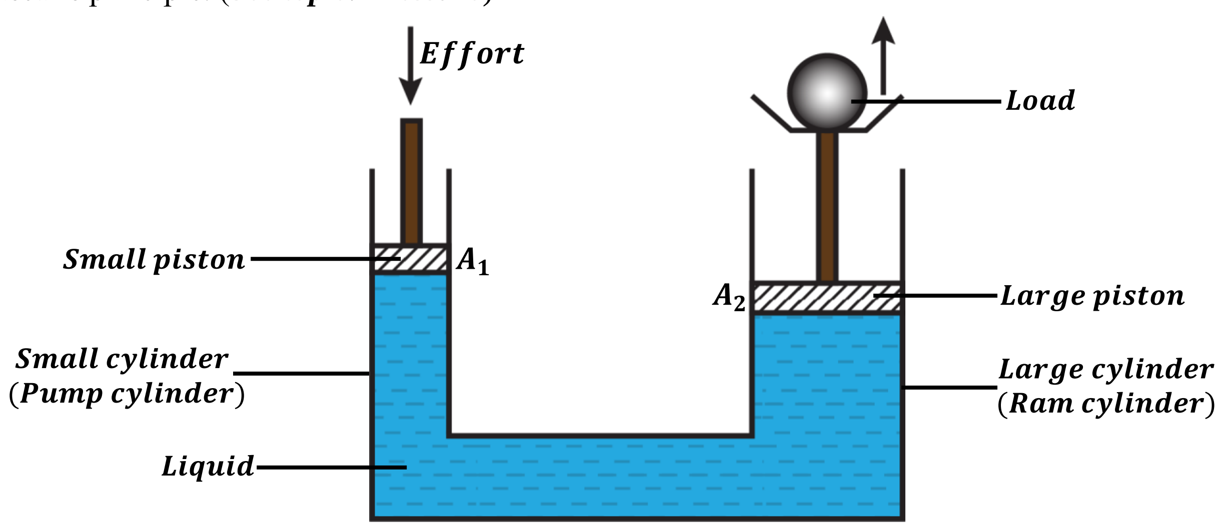
3. A screw jack is found to be 70% efficient. If an effort of 20N is used to lift a vehicle of 5000N and the pitch of the screw is 2mm. What is the length of the lever arm.

4. A screw of pitch 2.5cm is used to raise a load of 200kg when an effort of 50N is applied to the screw arm of length 20cm. Calculate;
   i) Mechanical advantage.
   ii) Velocity ratio.
   iii) Efficiency.

5. A Screw jack of pitch 2.5mm is operated by a force of 100N acting at a distance of 7cm from the axis about which the handle rotates and lifts a car weighing 792kg. calculate its efficiency.

HYDRAULIC PRESS OR LIFT

It's a device used to lift a large load using a small effort. Its operation is based on the knowledge of Pascal's principle. (***See topic: Pressure***)



Velocity ratio of a hydraulic press:

Let D_E be the distance moved by the small piston and A_1 be the area of the small piston.
Let D_L be the distance moved by the small piston and A_2 be the area of the large piston.

When an effort is applied on the small piston:

- The volume of the liquid pushed down by the small piston is equal to the volume of the liquid lifts up the large piston with the load.

RECALL: $Volume = Area \times Distance$

$$Volume\ of\ liquid\ pushed\ down\ small\ piston = Volume\ of\ liquid\ lifting\ large\ piston$$

$$A_1 \times D_E = A_2 \times D_L$$

$$\frac{D_E}{D_L} = \frac{A_2}{A_1}$$

- Since the pistons are circular, their areas equal to the area of a circle.

$$A_1 = \pi r^2, \qquad A_2 = \pi R^2$$

$$Then, \qquad \frac{D_E}{D_L} = \frac{A_2}{A_1} = \frac{\pi R^2}{\pi r^2}$$

Therefore;

$$\boxed{V.R = \frac{D_E}{D_L} = \frac{R^2}{r^2}}$$

Where R – *Radius of the large piston.*
r – *Radius of the small piston.*
D_E – *Distance moved by the effort.*
D_L – *Distance moved by the load.*

Examples:

1. The radius of the effort piston of a hydraulic lift is 1.4cm while that of the load piston is 7.0cm. This machine is used to raise a load of 1200N. Given that the machine is 80% efficient, calculate;
   a) Velocity ratio
   b) Mechanical advantage.
   c) Effort applied.

a) Velocity ratio

$$r = 1.4cm, \qquad R = 7cm$$

$$V.R = \frac{R^2}{r^2}$$

$$V.R = \frac{7^2}{1.4^2}$$

$$V.R = 25$$

b) Mechanical advantage

$$\eta = \frac{M.A}{V.R} \times 100\%$$

$$80\% = \frac{M.A}{25} \times 100\%$$

$$80\% \times 25 = 100\% M.A$$

$$M.A = \frac{2000\%}{100\%}$$

$$M.A = 20$$

c) Effort

$$M.A = \frac{L}{E}$$

$$20 = \frac{1200}{E}$$

$$E = \frac{1200}{20}$$

$$E = 60N$$

2. A hydraulic press is used to lift 400N using an effort of 20N. The diameter of the large cylinder is 100cm and the diameter of the small cylinder is 10cm. Find;

i) Velocity ratio

$$r = \frac{10}{2} = 5cm, \qquad R = \frac{100}{2} = 50cm$$

$$V.R = \frac{R^2}{r^2}$$

$$V.R = \frac{50^2}{5^2}$$

$$V.R = 100$$

ii) M.A

$$M.A = \frac{L}{E}$$

$$M.A = \frac{400}{20}$$

$$M.A = 20$$

iii) Efficiency

$$\eta = \frac{M.A}{V.R} \times 100\%$$

$$\eta = \frac{20}{100} \times 100\%$$

$$\eta = 0.2 \times 100\%$$

$$\eta = 20\%$$

3. A hydraulic machine has a ram cylinder (large cylinder) of diameter 30cm and a pump cylinder (small cylinder) of diameter 2cm. If the effort applied to the small piston is 70N and the efficiency of the machine is 80%, calculate the;
   a) Velocity ratio
   b) Mechanical advantage.
   c) Load lifted.

a) Velocity ratio

$$r = \frac{2}{2} = 1cm, \qquad R = \frac{30}{2} = 15cm$$

$$V.R = \frac{R^2}{r^2}$$

$$V.R = \frac{15^2}{1^2}$$

$$V.R = 225$$

b) Mechanical advantage

$$\eta = \frac{M.A}{V.R} \times 100\%$$

$$80\% = \frac{M.A}{225} \times 100\%$$

$$80\% \times 225 = 100\% M.A$$

$$M.A = \frac{18000\%}{100\%}$$

$$M.A = 180$$

c) Effort

$$M.A = \frac{L}{E}$$

$$180 = \frac{L}{70}$$

$$L = 180 \times 70$$

$$L = 12600N$$

EXERCISE:

1. A hydraulic machine has a large cylinder of 30cm and a small cylinder of 1cm. Given that the machine is 80% efficient and that the effort applied on the small piston is 50N, calculate;
   a) Velocity ratio
   b) Mechanical advantage.
   c) Maximum load that can be raised.

2. The area of the effort piston of a hydraulic lift is $56cm^2$ while that of the load piston is $224cm^2$. This machine is used to raise a load of 300kg through a height of 2.5mm. given that the machine is 75% efficient, calculate;
   a) Velocity ratio
   b) Mechanical advantage.
   c) Effort applied.
   d) Distance moved by the effort.

NOTE:

- If two simple machines are combined together, the overall velocity ratio is equal to the product of the individual velocity ratios of the two machines.

$$V.R = Velocity\ ratio\ of\ machine\ 1 \times Velocity\ ratio\ of\ machine\ 2$$